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Takaki, T.; Thies, M.

published in
Physical Review C
1988

DOI (link to publisher)
10.1103/PhysRevC.38.2230

document version
Publisher's PDF, also known as Version of record

citation for published version (APA)
Takaki, T., & Thies, M. (1988). One-nucleon knockout by pions and deltas. *Physical Review C*, *38*(5), 2230-2244. https://doi.org/10.1103/PhysRevC.38.2230

PHYSICAL REVIEW C VOLUME 38, NUMBER 5 NOVEMBER 1988

One-nucleon knockout by pions and deltas

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(Received 18 April 1988)

We present the first quantitative study of the $(\pi,\pi' N)$ coincidence reaction within the Δ-hole model and compare our results to recent Swiss Institute for Nuclear Research (SIN) and LAMPF data for ^{16}O near 240 MeV. A modified distorted-wave impulse approximation (DWIA), including medium corrections to the Δ propagator and containing no free parameters, reproduces the leading $(\pi^+,\pi^+ p)$ channel for p-shell knockout satisfactorily, provided we use shell-model spectroscopic factors. The weak $(\pi^-,\pi^- p)$ and $(\pi^+,\pi^0 p)$ channels are found to be sensitive to interference effects between the DWIA mechanism and knockout induced by ΔN collisions. This latter two-step process is also calculated microscopically, using a simple form for the $\Delta - N$ interaction guided by pion absorption. Our model qualitatively confirms earlier estimates, but cannot reproduce the isospin ratios over the whole kinematical range. In particular, it systematically underestimates the charge-exchange cross sections by 30%.

I. INTRODUCTION

The most conspicuous feature of the pion-*nucleon* interaction at intermediate energies is the dominance of the Δ resonance. It is well established by now that the Δ also plays a crucial dynamical role in pion-*nucleus* reactions.[1–10] This has gradually shifted the emphasis from the behavior of the pion inside nuclei to that of the Δ resonance. Studying the Δ-nucleus interaction is of interest as a prime example of the behavior of a short-lived, excited hadron in nuclear matter. Moreover, it allows to unify the description of nuclear reactions induced by different probes, notably pions and photons,[11] at corresponding energies.

Former investigations of the Δ-nucleus dynamics can roughly be divided into two phases. In the first phase,[1–7] the global characteristics of the Δ-nucleus interaction have been determined. In the second phase,[12,13] the more difficult problem of relating these characteristics to the underlying Δ-nucleon interaction is being tackled. Whereas phase I can be considered as essentially completed, a lot remains to be done on the second, more ambitious problem. Our paper is an attempt to contribute to this particular issue.

As far as the gross features are concerned, the most successful approach has been a combination of microscopic and phenomenological tools through the Δ-hole model.[1–7] Since this model has bee discussed many times in the literature, we shall only briefly recall its most important features. So far, theory has been able to treat elastic π-nucleus scattering microscopically in the 1p-1h space, without unjustified technical approximations. This is equivalent to a finite nucleus g-matrix calculation, and involves a careful treatment of nonstatic effects and Fermi motion (necessary to account for Δ propagation), Pauli blocking of the Δ width, and binding corrections on the level of the shell model. On the other hand, the coupling to more complicated channels (2p-2h , π-2p-2h, etc., usually referred to as Q space) and possible direct "bare Δ"-nucleon interactions have been treated phenomenologically by simply adding a term to the Δ self-energy, the so-called "spreading potential."[3,5,7] This coupling to many-particle, many-hole states is particularly important for pions due to their high absorption probability. The most widely used spreading potential has been parametrized in terms of a central and spin-orbit potential, and involves two complex parameters.[7] Such a form describes adequately total and elastic scattering cross section for ^{4}He, ^{12}C, and ^{16}O, throughout the resonance region. An important confirmation of this approach are recent observations that also heavier nuclei (^{40}Ca, Ref. 14) and open-shell nuclei (^{6}Li, Ref. 15) yield similar strength parameters.

The Δ-hole model has passed successfully several tests which demonstrate its predictive power. Assuming that the dominant damping mechanism is absorption, one can estimate total absorption cross sections, and they agree well with the large cross sections seen experimentally.[3,5,7] A detailed study of the inclusive (π,π') reaction on ^{4}He and ^{16}O allowed to reproduce the energy-loss spectra of the pion over a large kinematical region.[9,10] Thus, the bulk of the π-nucleus reaction cross section is well accounted for by the Δ-hole model. Finally, the same Δ-nucleus interaction parameters can explain neutral photo-pion production data,[16,17] confirming the basic Δ-hole "doorway" hypothesis in a nontrivial way.

Let us now turn to the second phase of investigations, where one tries to go beyond characterizing the Δ-

nucleus interaction by a spreading potential. Here, some developments have taken place over the last few years. They were triggered by the observation that isospin ratios in pion-nucleus reactions are sometimes significantly different from the free ones. Striking examples are the excitation of the $T=0/T=1$ doublet of 1^+ states in ^{12}C,[18] or the comparison of inclusive (π^+,π^+) and (π^+,π^0) on ^{16}O.[19] In these $T=0$ nuclei, deviations from free isospin ratios tell us at once that there must be reaction mechanisms not accounted for by the conventional distorted-wave impulse approximation (DWIA), irrespective of all the details of the Δ nucleus potential. This in turn suggests to seek for an explanation which involves the basic Δ-N interaction. Along these lines, Hirata *et al.*[12] pointed out that if the Δ experiences a strong potential, it can also excite the nucleus through a ΔN collision. Such processes may have a different isospin dependence and can interfere with the DWIA-type mechanism. Assuming that the Δ-N interaction is dominated by the $^5S_2$ $(T_{\Delta N}=1)$ channel (as suggested by the $\pi^+d \rightarrow pp$ reaction), one can make various qualitative predictions which seem to agree with the trends of the experimental data. A quantitative study of discrete excitations in ^{12}C by Takaki[13] essentially confirmed the results of Ref. 12, but also revealed the necessity to include ΔN channels not related to absorption.

The cleanest way to test these ideas and try to get more detailed information about the Δ-N interaction is presumably by means of the quasifree one-nucleon knockout reaction $(\pi,\pi'N)$. Here, the Δ-N interaction induced process is predicted to interfere constructively in charge exchange, destructively in the weak (π^-,π^-,p) channel, and to have little effect on the strong (π^+,π^+p) channel.[12] Coincidence data for all three isospin channels on ^{16}O near 240 MeV have recently been obtained at SIN $(\pi^\pm,\pi^\pm p)$ (Refs. 20 and 21) and LAMPF $(\pi^+,\pi^0 p)$.[22] These measurements have led us to perform a quantitative Δ-hole calculation of the $(\pi,\pi'N)$ reaction. We shall address two main questions: How well can we understand the absolute magnitude and kinematical dependence of the coincident cross sections? And what can we learn from a detailed study of the isospin ratos? For the first part, the results of Refs. 9 and 10 suggest a focus on the (π^+,π^+p) channel, which should be adequately described in a modified DWIA, along the lines used previously for the inclusive (π,π') reaction. To answer the second question, it is evident that we have to go beyond the DWIA framework. These two issues are reflected in the organization of the paper. In Sec. II we collect all the formalism necessary to explain what we have done. In Sec. III, we discuss our results from a modified DWIA calculation, and in Sec. IV we include explicitly proton knockouts induced by the Δ-N interaction. Finally, Sec. V contains a brief summary and our main conclusions.

II. THEORY OF THE EXCLUSIVE $(\pi,\pi'N)$ REACTION

A. Kinematics and cross section

The quasifree knockout of a proton by a pion proceeds dominantly through a Δ excitation, as illustrated in Fig. 1. A pion with three momentum $\mathbf{k}_L$ and energy ω_L is scattered to a final state $\mathbf{k}'_L$ and ω'_L. The nucleus makes a transition from the ground state to the final state consisting of the knocked-out proton with momentum $\mathbf{p}'_L$ and energy E'_L and the recoiling residual nucleus with $\mathbf{p}_R$ and $E_R=T_R+M_{A-1}+E_X$, where T_R is the kinetic energy, M_{A-1} the rest mass, and E_X the excitation energy.

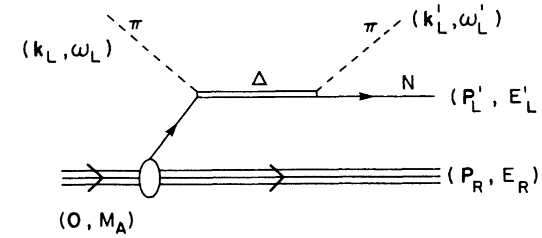

FIG. 1. Basic mechanism for the $(\pi,\pi'N)$ reaction through Δ. Kinematical variables refer to the lab frame.

In the coincidence experiment, final momenta and energies of pion and proton $\mathbf{k}'_L,\omega'_L,\mathbf{p}'_L,E'_L$ are measured and so the final state of the recoiling residual nucleus is uniquely fixed by momentum and energy conservation. As one can also specify the state of the Δ and struck nucleon, such a kinematically complete experiment provides important information on Δ propagation in the nuclear medium as well as the bound nuclear wave function.

Generally, the reaction cross section at a fixed excitation energy of the residual nucleus in the laboratory (lab) frame can be written as

$$\frac{d^5\sigma}{dT_\pi d\Omega_\pi d\Omega_p}=K\sum_{\mu f_x}|\langle \mathbf{k}'\mathbf{p}'\mu;f_x\,|\,T_{ACM}\,|\,\mathbf{k};0\rangle|^2 . \quad (1)$$

K is a kinematical factor in which the recoil energy is neglected,

$$K=\frac{k'_L p'_L \omega\omega' E'}{(2\pi)^5 k_L} , \quad (2)$$

with k,ω,k',ω' the initial (final) pion momenta and energies, p',E' the knocked-out proton momentum and energy in the center of mass (c.m.) frame. Labels μ and f_x denote the spin of the proton and a specific final state of the residual nucleus with excitation energy E_X. The amplitude T_{ACM} is evaluated in the c.m. frame.

To compare theory with experiment, one has to include the finite solid angles of the detectors in the calculation of cross sections as follows:

$$\frac{d^5\bar{\sigma}}{dT_\pi d\Omega_\pi d\Omega_p}=\frac{\int_{\Delta\Omega_p}d\Omega_p\int_{\Delta\Omega_\pi}d\Omega_\pi\frac{d^5\sigma}{dT_\pi d\Omega_\pi d\Omega_p}}{\int_{\Delta\Omega_p}d\Omega_p\int_{\Delta\Omega_\pi}d\Omega_\pi} . \quad (3)$$

We note that because of rather wide solid angles of the detectors, e.g., $\Delta\theta_p=15°$, $\Delta\phi_p=50°$ for the SIN data[20] which will be shown later, the behavior characteristic of the p-shell momentum distribution is lost in the energy spectra of the pion, and the average cross section

$d^5\bar{\sigma}/dT_\pi d\Omega_\pi d\Omega_p$ is decreased by about 50% at backward pion scattering angle, compared to the nonaveraged cross section $d^5\sigma/dT_\pi d\Omega_\pi d\Omega_p$ calculated at the quasifree angle. Our theoretical predictions in this paper are the average cross sections.

B. Brief review of the Δ-h model

In this section we review the main physical ideas behind the Δ-hole model[1-7] and provide the necessary formalism, following closely the approach developed in Refs. 3, 5, and 7. Starting point is an isobar model for πN scattering. In this model, the πN scattering amplitude is given in a separable form,

$$t_{\pi N}=F\frac{1}{D(E)}F^{\dagger} . \tag{4}$$

Here, F and $F^{\dagger}$ are vertex functions for the $\pi\Delta N$ coupling, and the D function, $D(E)$, represents the inverse Δ propagator in free space which has the usual resonance form:

$$F=g_{\pi\Delta N}\frac{\mathbf{S}\cdot\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2+a^2}\mathbf{T}_\alpha , \tag{5a}$$

$$D(E)=E-E_R(E)+i\Gamma(E)/2 . \tag{5b}$$

E and $\boldsymbol{\kappa}$ are the total energy and relative momentum of the πN system in the c.m. frame. The vertex function has a simple p-wave monopole form with a cutoff mass $a=300$ MeV. The operators $\mathbf{S}$ and $\mathbf{T}$ are transition spin and isospin operators which connect spin $\frac{3}{2}$ and $\frac{1}{2}$ or isospin $\frac{3}{2}$ and $\frac{1}{2}$. All quantities in Eq. (5) are determined by the πN phase shift in the P33 channel.

The amplitude (4) is defined in the πN c.m. frame. To extend the isobar model to the nuclear system, the amplitude must be transformed to a general frame. Neglecting all medium corrections except Fermi motion, the following amplitude describes πN scattering in the nuclear medium:

$$T_{\pi A}=\sum_i F_i\frac{1}{D(E-T_\Delta)}F_i^{\dagger} . \tag{6}$$

Here, T_Δ is the kinetic energy of the Δ, and $E-T_\Delta$ the πN c.m. energy. The subscript of the vertex functions labels the ith nucleon.

Now we are ready to construct the amplitude for the nuclear system including medium corrections. Let us first consider elastic scattering on closed-shell nuclei. We expect that the nuclear amplitude has the same general form as the πN isobar model amplitude, but that all nucleons in the nucleus contribute to the scattering. Thus the amplitude is written as

$$T_{\pi A}=\sum_{ij} F_i G_{\Delta h}F_j^{\dagger} . \tag{7}$$

This amplitude is illustrated in Fig. 2. $G_{\Delta h}$ is the Δ-h propagator which includes all medium effects of the Δ in the nucleus. We consider the operator in the Δ-nucleus space instead of the pion-nucleus space and add the medium corrections to the delta-hole propagator step by step.

First we take into account Fermi motion, the shell-

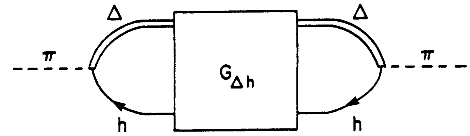

FIG. 2. Diagrammatic representation of the elastic π-nucleus amplitude in the Δ-hole model.

model potential V_B, and the hole energy, H_{A-1}. The Δ-h propagator is then written as

$$G_{\Delta h}^{-1}=D(E-H_\Delta) , \tag{8a}$$

with

$$H_\Delta=T_\Delta+V_B+H_{A-1} . \tag{8b}$$

T_Δ is the kinetic energy operator. The modification due to V_B and H_{A-1} can be considered as an energy shift of the Δ resonance.

The propagator of (8) still includes decay of the Δ into Pauli-violating states. So one has to subtract this contribution, i.e., the "Fock term," from Eq. (8):

$$G_{\Delta h}^{-1}=D(E-H_\Delta)-W_p , \tag{9a}$$

with

$$W_p=-\sum_i F_i^{\dagger}G_\pi P_V F_i . \tag{9b}$$

Here G_π is the free pion Green's function, and P_V is the projection operator onto Pauli-violating states.

So far we considered only one-body-type corrections, diagonal in the hole states. Pion multiple scattering is very important in the energy range in which we are interested here. It can be included by means of a Δ-h interaction,

$$G_{\Delta h}^{-1}=D(E-H_\Delta)-W_p-W_\pi , \tag{10a}$$

with

$$W_\pi=\sum_{i\neq j} F_i^{\dagger}G_\pi P_0 F_j . \tag{10b}$$

Here P_0 is the projection operator onto the nuclear ground state. The rescattering operator W_π corresponds to a (retarded) one-pion exchange potential. The generalized amplitude $T_{\pi A}$ with the propagator of (10), which is equivalent to a g-matrix calculation of the optical potential model, can be evaluated numerically in the Δ-h space without any further approximations, and involves no free parameters. However, such a calculation overestimates the total cross section and integrated elastic cross section of pion-nucleus scattering significantly. Clearly there must be an important physical process which is not included in Eq. (10) and has a strong damping effect.

We have left out the coupling to complicated many-particle–many-hole states. To account for this effect, a phenomenological "spreading potential"[3,5,7] has been introduced:

$$G_{\Delta h}^{-1}=D(E-H_\Delta)-W_p-W_\pi-V_{sp} , \tag{11a}$$

with

$$V_{sp} = V_0 \rho(r)/\rho(0) + 2W_0 \mathbf{L}_\Delta \cdot \mathbf{S}_\Delta v_{LS}(r) . \tag{11b}$$

Equation (11) is the full Δ-h propagator in the Δ-h model. The strength parameters V_0 and W_0 are adjusted to reproduce the total and elastic cross sections. For example,

$$\begin{aligned} V_0 &= 19 - i47 \text{ MeV} , \\ W_0 &= -10 - i4 \text{ MeV} , \end{aligned} \tag{12}$$

for the ^{16}O nucleus in the Δ region.[7] An important feature is the large imaginary part of the central potential. One of the purposes of this paper is to investigate the physical origin of the spreading potential. Some hints have already been obtained in previous studies,[1-7] where it was suggested that the large imaginary component arises from the coupling to the pion absorption channel.

Before applying the above formalism to the $(\pi,\pi' p)$ reaction, we have to refine the treatment of pion absorption via the spreading potential. Following Ref. 12, we introduce the Δ-N interaction, which induces a new reaction mechanism. The spreading potential is assumed to arise from the effective two-body Δ-N interaction $\tilde{t}_{\Delta N}$ as shown in Fig. 3. The general Δ-N interaction $\tau_{\Delta N}$ is defined as

$$\tau_{\Delta N} = W_\pi + \tilde{t}_{\Delta N} \tag{13}$$

$$= v_{\Delta N} + v_{\Delta N} G Q \tau_{\Delta N} . \tag{14}$$

Here Q is the projection operator to Q space and G is the free Δ-N Green's function. The potential $v_{\Delta N}$ includes the one-pion exchange, pion absorption, etc., as illustrated in Fig. 4. In this description, the Δ-h propagator is built from the one-body operator $D(E-H_\Delta)-W_p$ and the two-body operator $\tau_{\Delta N}$,

$$G_{\Delta h}^{-1} = D(E-H_\Delta) - W_p - \tau_{\Delta N} . \tag{15}$$

The above extended form of the delta-hole propagator can be applied to inelastic as well as to elastic scattering. In the next section we will discuss the implications for the $(\pi,\pi N)$ reaction.

C. Transition amplitude for the $(\pi,\pi' N)$ reaction

The most important mechanism in the $(\pi,\pi' N)$ reaction is the one step process arising from the elementary πN interaction as shown in Fig. 5(a). This one-body process is the leading term in the sense of a density expansion. Perturbatively, the second-order process is induced

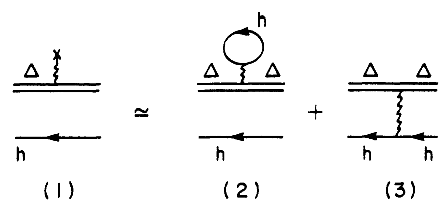

FIG. 3. Schematic identification of the spreading potential (1) with the Δ-N interaction (2) and (3).

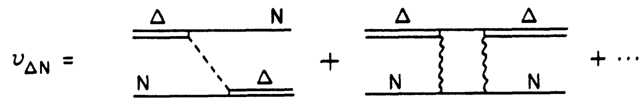

FIG. 4. Driving terms of the Δ-N interaction, Eqs. (13) and (14).

by the Δ-N interaction, Fig. 5(b). We consider the knock-out process on a closed shell target. The nuclear transition to first order between the ground state $|0\rangle$ and the particle hole state $|\mathrm{ph}\rangle$ is illustrated in Fig. 6, which corresponds to the matrix element of the process Fig. 5(a), and self-energies are inserted into each particle line. This diagram will be called modified DWIA.[8-10] The transition amplitude for the modified DWIA is written as

$$f_{\mathrm{I}} = \langle \psi_{\mathbf{k}'}^{(-)} \phi_{\mathbf{p}}^{(-)} ; h \,|\, \sum_{ij} F_i \hat{G}_{\Delta h} F_j^\dagger \,|\, \psi_{\mathbf{k}}^{(+)} ; 0 \rangle , \tag{16}$$

with

$$\hat{G}_{\Delta h}^{-1} = D(E-H_\Delta) - W_p - \tilde{t}_{\Delta N} . \tag{17}$$

Here $\psi_{\mathbf{k}}^{(+)}$ and $\psi_{\mathbf{k}'}^{(-)}$ are initial (final) distorted pion wave functions calculated by the Δ-h model,

$$|\psi_{\mathbf{k}}^{(+)}\rangle = (1 + G_\pi T_{00}) |\mathbf{k}\rangle , \tag{18}$$

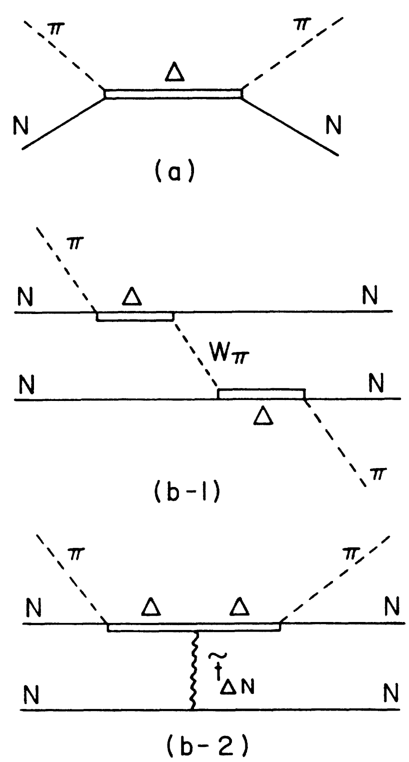

FIG. 5. Elementary processes contributing to the $(\pi,\pi' N)$ reaction (a) first-order process caused by πN scattering, (b) second-order process induced by the Δ-N interaction: (b-1) one pion exchange, W_π and (b-2) effective Δ-N interaction, $\tilde{t}_{\Delta N}$.

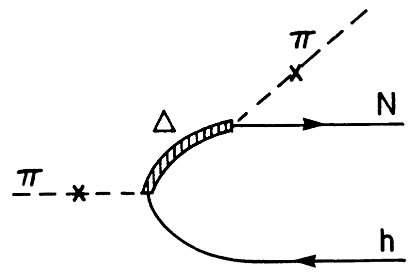

FIG. 6. Diagram for modified DWIA. The Δ-hole propagator (hatched) and the pion wave function (crossed) are defined by Eqs. (17) and (18), respectively.

where T_{00} is the elastic π-nucleus T-matrix. The proton wave function $\phi_{\rm p}^{(-)}$ is calculated from a phenomenological optical potential. Up to this point there is no free parameter because the self-energies of pion and delta are determined by the analysis of elastic scattering. It is instructive to see how far the modified DWIA can predict the $(\pi,\pi' N)$ reaction. This is one of our aims in this paper. Later on we will discuss in detail our numerical results (Sec. III).

Because of the simple isospin structure, one can easily obtain the isospin ratio of the cross section of (π^+,π^+p), (π^-,π^-p), and (π^+,π^0p) by assuming Δ dominance. The amplitudes are written as

$$
\begin{aligned}
&f_{\rm I}(\pi^+ - \pi^+ pp^{-1}) = f_{R_0}\,,\\
&f_{\rm I}(\pi^- \rightarrow \pi^- pp^{-1}) = \tfrac{1}{3} f_{R_0}\,,\\
&f_{\rm I}(\pi^+ \rightarrow \pi^0 pn^{-1}) = -\frac{\sqrt{2}}{3} f_{R_0}\,.
\end{aligned}
\tag{19}
$$

Consequently the ratio $\sigma(\pi^+,\pi^+p)$:$\sigma(\pi^-,\pi^-p)$: $\sigma(\pi^+,\pi^0p)$ is 9:1:2, like in free πN scattering. Clearly recent experiments, which show significant deviation from the free ratios,[20-22] are not reproduced even by the modified DWIA, and a refinement of theory is called for.

To solve this difficulty, one has to take into account the second-order process induced by the Δ-N interaction. There are two kinds of interaction: one-pion exchange which corresponds to the rescattering operator W_π, and the effective Δ-N interaction $\tilde{t}_{\Delta N}$ which is related to the spreading potential [see Fig. 5(b)]. To second order, the two-body operators of Fig. 5(b) give rise to the nuclear transition matrix elements illustrated in Fig. 7. We note that diagrams (7.1)–(7.3) and (7.5)–(7.6) are already included in the amplitude of the modified DWIA. These diagrams correspond to self-energies of the delta or the pion, namely, initial pion distortion (7.1), final pion distortion (7.2), Fock term (7.3), Δ-self-energy (7.5), and Δ-h vertex correction (7.6). Since the Δ-h vertex correction renormalizes the Δ-self-energy,[12,13] these two contributions can be effectively described by the spreading potential. The remaining diagrams (7.4), (7.7), and (7.8) cannot be renormalized into the modified DWIA amplitude and therefore are genuine two-body processes. As diagram

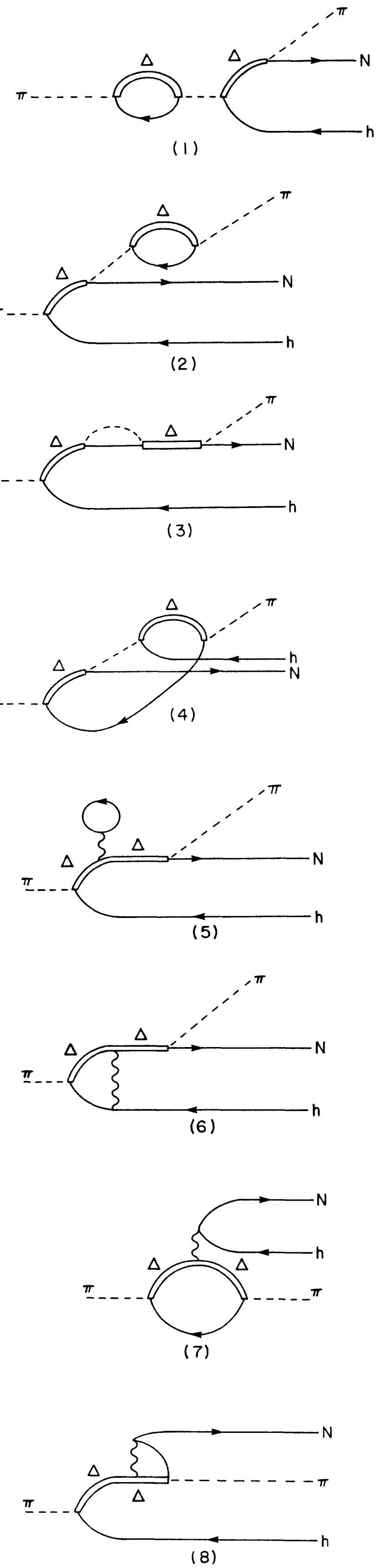

FIG. 7. Diagrams for the second-order processes induced by one pion exchange [(1) ∼4] and the effective Δ-N interaction [(5) ∼8].

(7.4), the effects of which are discussed in the Appendix, has the same isospin structure (7.7) in a given ΔN isospin channel, we shall consider only diagrams (7.7) and (7.8). They are called direct term and exchange term, respectively.

The transition amplitude for these second-order process is written as

$$f_{\rm II}=\langle \psi_{\mathbf{k}'}^{(-)},\phi_{\mathbf{p}}^{(-)};h \mid \sum_{ij} F_i G_\Delta^{(2)}\tilde{t}_{\Delta N}\hat{G}_{\Delta h}F_j^\dagger \mid \psi_{\mathbf{k}}^{(+)};0\rangle \ , \quad (20)$$

where $G_\Delta^{(2)}$ is the Δ propagator in the Δp-2h states. The detailed derivation of Eq. (20) is given in Ref. 13. As is indicated in Eq. (20), the self-energies for pion and delta are included in diagrams (7.7) and (7.8), like in the modified DWIA. The Δ-N interaction consists of isospin $T_{\Delta N}=1$ and $T_{\Delta N}=2$ components. One can obtain the isospin dependence of the transition amplitude for the $(\pi,\pi' N)$ reaction as follows:

$$f_{\rm II}(\pi^+\to\pi^+ pp^{-1})=\tfrac{1}{12}f_D(T_{\Delta N}=1)+\tfrac{1}{4}f_E(T_{\Delta N}=1)+\tfrac{5}{4}[f_D(T_{\Delta N}=2)-f_E(T_{\Delta N}=2)] \ ,$$

$$f_{\rm II}(\pi^-\to\pi^- pp^{-1})=\tfrac{1}{3}\{\tfrac{11}{4}f_D(T_{\Delta N}=1)+\tfrac{1}{4}f_E(T_{\Delta N}=1)+\tfrac{5}{4}[f_D(T_{\Delta N}=2)-f_E(T_{\Delta N}=2)]\} \ , \quad (21)$$

$$f_{\rm II}(\pi^+\to\pi^0 pn^{-1}) =-\frac{\sqrt{2}}{3}\{-\tfrac{5}{4}f_D(T_{\Delta N}=1)+\tfrac{1}{4}f_E(T_{\Delta N}=1)+\tfrac{5}{4}[f_D(T_{\Delta N}=2)-f_E(T_{\Delta N}=2]\}.$$

Here f_D is the amplitude of the direct term and f_E the amplitude of the exchange term. These amplitudes f_D and f_E do not include the isospin operator. Isospin factors have been spelled out explicitly. From the general expression, Eq. (21), one can recognize that the $T_{\Delta N}=2$ component has the same effect on all three reactions. This is simply due to the fact that if the πNN state has total isospin 2, the pion and the knocked-out nucleon must be in a $T=\frac{3}{2}$ state, just like in the DWIA. Therefore, deviations from the free isospin ratio can only be generated through the $T_{\Delta N}=1$ component. Experimental data indicate the importance of the $T_{\Delta N}=1$ Δ-N interaction.

As mentioned before, the spreading potential is assumed to arise mainly from pion absorption. In this case, the effective Δ-N interaction $\tilde{t}_{\Delta N}$ represents the two-nucleon absorption via $\Delta N \rightleftharpoons NN$. Based on the fact that the $^5S_2$ $(T_{\Delta N}=1)$ channel of the Δ-N interaction is dominant, we assume, in the following, simple zero-range form:

$$\tilde{t}_{\Delta N}=C_{21}\delta(\mathbf{r}_\Delta-\mathbf{r}_N)P_{S_{\Delta N}=2}P_{T_{\Delta N}=1} \ , \quad (22)$$

where $P_{S_{\Delta N}=2}$ and $P_{T_{\Delta N}=1}$ are spin and isospin projection operators. Using the strength of the central potential of Eq. (12) and comparing the isobar doorway expectation value of the spreading potential and the Δ-N interaction,[12,13] we find

$$C_{21}=2.17-i5.31 \text{ fm}^2 \ . \quad (23)$$

III. DWIA RESULTS

The modified DWIA calculation for the $(\pi,\pi' p)$ exclusive reaction has been performed essentially in the same way as the calculation for the (π,π') inclusive reaction,[10] using the same input parameters except for the proton-nucleus optical potential . Clearly, the full optical potential with the imaginary part has to be used in the exclusive case. We have employed the phenomenological proton-nucleus optical potential of Comfort and Karp[23] which includes a spin-orbit term and is determined from p-^{12}C scattering data between kinetic energies of 10 and 200 MeV. It was found that their potential provides a good description of p-^{16}O scattering for proton kinetic energies relevant to our calculations. Their parameters at 135 MeV are similar to those found by Kelly.[24]

In our calculation, the nonresonant πN interaction and the π and Δ Coulomb interaction are included in the pion wave functions and the transtion operator, in addition to the resonant term. For detailed discussions on the pion wave function and the transition operator for the DWIA within the Δ-h approach, we refer the reader to Ref. 10. The separation energies used are 12 and 18 MeV for the $p_{1/2}$ and $p_{3/2}$ shells, respectively, and 40 MeV for the $s_{1/2}$ shell. The hole-width was not taken into account in our calculation. The bound nucleon wave functions are harmonic oscillator wave functions with the size parameter determined from elastic electron scattering. The spectroscopic factor N_α is assumed to be 1.0, which means pure single-particle transition (i.e., $|0\rangle \to |\text{ph}\rangle$).

It is instructive to see the difference in the distortion between pion and proton in the nucleus. Following Ref. 10, we illustrate the quantity $\rho(r)=|\Psi(r)|^2$ in Fig. 8, where $\Psi(\mathbf{r})$ is the pion or proton continuum wave function. $\rho(\mathbf{r})$ represents the probability density for finding a particle at point $\mathbf{r}$ in the target. $\rho(\mathbf{b},z)$ is plotted for fixed impact parameter ($b=1$ fm) as a function of z. The pion wave function at 240 MeV is strongly damped inside the nucleus. However, the proton is able to deeply penetrate the target, because it has a long mean free path of ~ 5 fm.

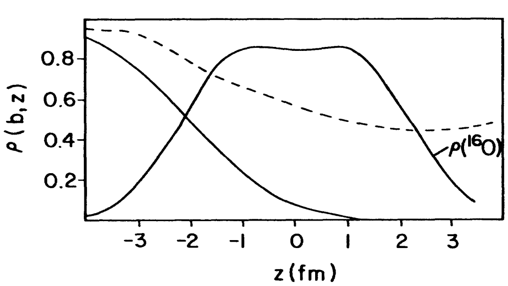

FIG. 8. Absolute square of three-dimensional distorted pion (solid curve) and proton (dashed) wave functions, plotted as a function of z for fixed impact parameter $b=1$ fm. The incident pion (proton) energy is 240 (150) MeV. The profile of the ^{16}O matter density at $b=1$ fm is also shown, in arbitrary units. The curve for the pion is taken from Ref. 10.

Due to the strong pion "absorption," the $(\pi,\pi'p)$ reaction takes place dominantly on the surface of the nucleus and is less sensitive to the proton behavior in the inner region of the nucleus, as compared to the $(e,e'p)$ or $(p,2p)$ reactions.

Before confronting our results in the modified DWIA with data, some remarks about kinematical and experimental conditions are necessary. Data were taken at several pion angles with the proton detector near the corresponding quasifree angle. $(\pi^{\pm},\pi^{\pm}p)$ cross sections were measured by Kyle *et al.*[20,21,25] at SIN. In this experiment, the energy resolution of the pion ($\sim$4 MeV) is good enough to resolve the $p_{1/2}$ and $p_{3/2}$ shell proton removal, although there remains some uncertainty in the cut for backward scattered pions. As mentioned before, the proton detector has a finite solid angle, i.e., $\Delta\theta_p = 15°$ and $\Delta\phi_p = 50°$. Protons with energy less than 35 MeV are not detected. So for the high pion energy side of the energy spectra, the comparison between the calculation and the data has to be done with care. On the other hand, in the (π^+,π^0p) experiment measured by Gilad *et al.*[22] at LAMPF, two dominant peaks corresponding to the p-shell removal could not be separated because of insufficient energy resolution for the π^0 ($\sim$6 MeV), and so the summed cross section for the p-shell removal is compared with the calculation. For the charge-exchange reaction we have taken into account the finite solid angle of the pion detector (i.e., $\Delta\theta_\pi = 10°$ and $\Delta\phi_\pi = 10°$).

In Figs. 9–11, we present our results for the (π^+,π^+p)

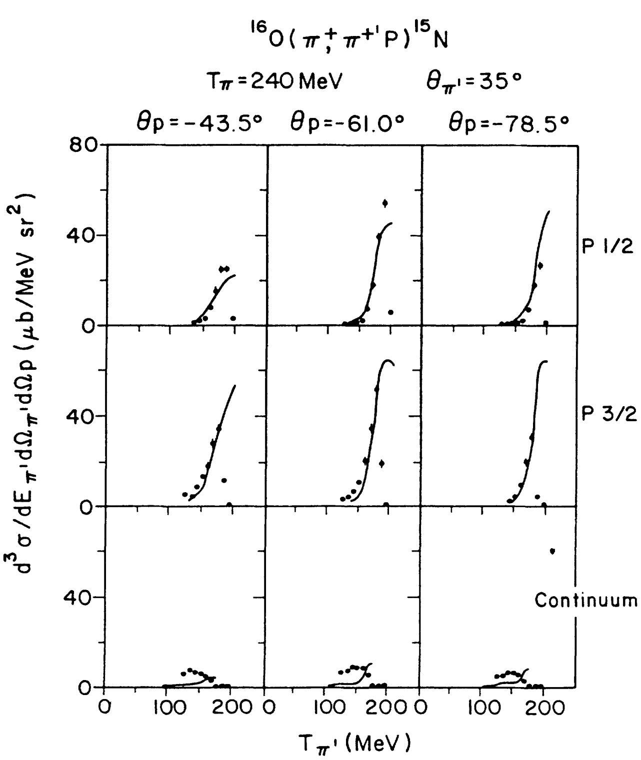

FIG. 9. Differential cross sections for $^{16}O(\pi^+,\pi^+p)^{15}N$ at 35° pion scattering angle for each of the proton telescopes, and regions of ^{15}N excitation energy corresponding to $p_{1/2}$ shell removal, $p_{3/2}$ shell removal, and continuum final state. Solid curves: modified DWIA calculation. Data from Refs. 21 and 25.

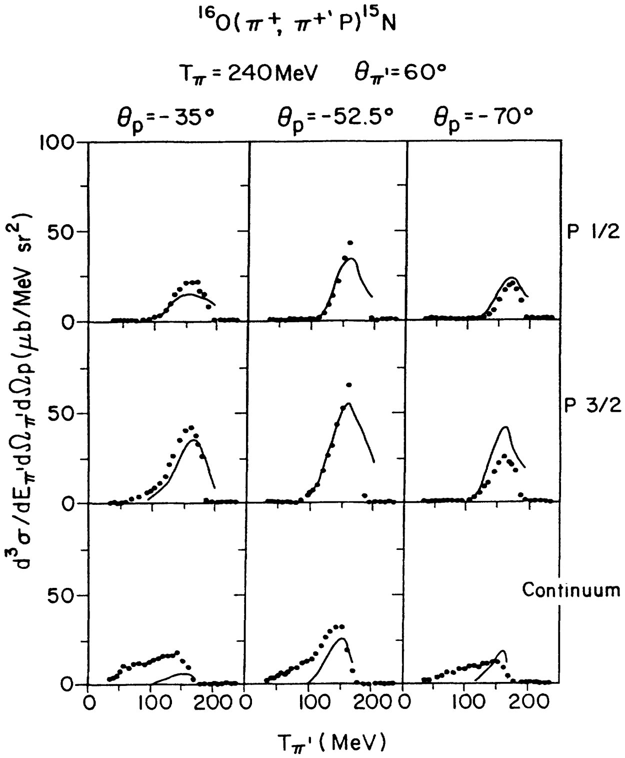

FIG. 10. Same as Fig. 9, but 60° pion scattering angle.

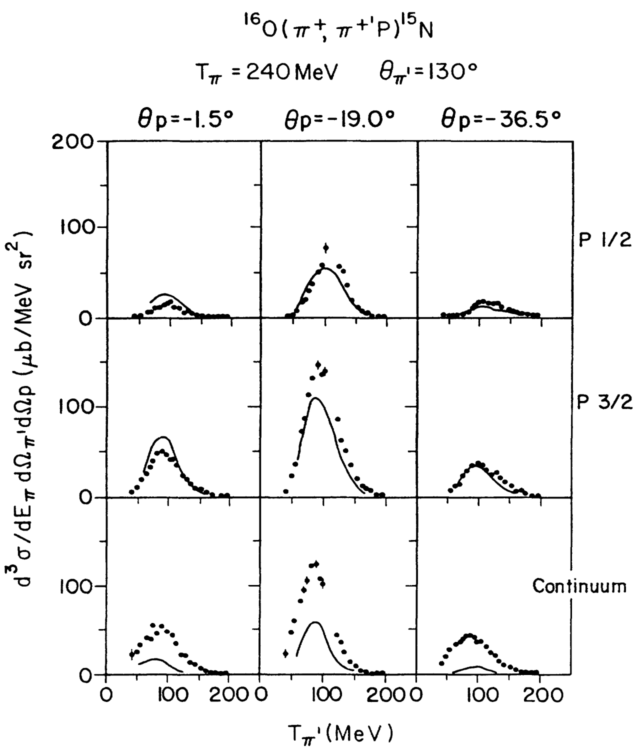

FIG. 11. Same as Fig. 9, but 130° pion scattering angle.

reaction on ^{16}O at pion angles of 35°, 60° and 130°, respectively, together with the SIN data. Each figure contains the differential cross sections for three proton angles, the central value being the quasifree angle. Furthermore, they are divided into three regions of the excitation energy, corresponding to the $p_{1/2}$ shell removal ($E_X < 4$ MeV), the $p_{3/2}$ shell removal (4 MeV $< E_X <$ 15 MeV), and the continuum part ($E_X >$ 15 MeV) from top to bottom, respectively. In the region of the $p_{1/2}$ shell removal, only the ^{15}N ground state contributes to the cross section; in the region of the $p_{3/2}$ shell removal, the 6.3 MeV excited state has a strong peak, but the 10.7 MeV excited state and some other complex states also contribute to the cross section. The continuum part presumably consists of both the $s_{1/2}$ shell removal and multiparticle emission. Each figure shown is the differential cross section defined in Eqs. (1) and (2) as a function of the emitted pion energy. In our calculation, the contribution of multiparticle emission is not taken into account. This may be the reason why we underestimate the continuum part, as shown in the figures. In the region of the quasielastic peak, the multiparticle emission in the (π^+,π^+p) reaction is expected to be mainly due to final-state interactions. This can be inferred from the theoretical analysis[10] of the inclusive (π,π') reaction, where the pion single scattering process (corresponding to the DWIA) has been found to be the dominant mechanism.

In the following, we shall concentrate on the p-shell removal where a comparison between theory and experiment is more meaningful. The shape of the cross section is well described by the theory. It does not show the characteristic p-shell distribution (i.e., a double hump) because of the finite angle acceptance of the proton detector; indeed, the calculated nonaverage cross section has such a p-shell shape, particularly at backward pion angles. For the smaller pion angles (35° and 60°), the theory is in good agreement with the experiment, except for the $p_{3/2}$ shell removal at $\theta_\pi = 60°$ and $\theta_p = -70°$. The calculation is too high at this particular set of angles. One would expect that the cross section for the $p_{3/2}$ shell removal is two times larger than the $p_{1/2}$ removal, if the spin dependent interaction is weak. This expectation is fulfilled for both the calculation and the data at the other angles. From this point of view, it seems difficult to understand the discrepancy at this angle within the DWIA approach.

At the largest pion angle (130°), the theory reproduces the experiment satisfactorily as far as the $p_{1/2}$ shell removal is concerned. We note that the original data of the $(\pi^\pm,\pi^\pm p)$ reaction are contaminated by hydrogen due to the use of a water target, and therefore the corrected data shown in the above figures contain some ambiguities around the quasifree peak. In fact, there are no data points near the quasifree peak, and the highest data point for the $p_{1/2}$ shell removal in Fig. 11 should probably be disregarded.[25] On the other hand, for the $p_{3/2}$ shell removal, the calculation underestimates the data significantly at the quasifree angle $\theta_p = -19°$. The ratio between the $p_{3/2}$ and $p_{1/2}$ shell removal is close to 3 for the data but 2 for the calculation. However, it is difficult to draw definite conclusions about the results of the $p_{3/2}$ shell removal at present, since there is some additional uncertainty in the cut between the $p_{3/2}$ shell removal and the continuum region.

Our DWIA calculations for the (π^-,π^-p) reaction at three angles are presented in Figs. 12–14 as a dashed line, together with the SIN data. The solid lines in the figures will be discussed in the next section. Like in the results for the (π^+,π^+p) reaction, the strength of the continuum part cannot be explained by the $s_{1/2}$ shell removal only. From the comparison between the data of (π^+,π^+p) and (π^-,π^-p), it is found that the ratios of the cross sections for the multiparticle emission to the single nucleon knockout is larger in the case of the (π^-,π^-p). This indicates the presence of some additional isospin-dependent reaction mechanism. The continuum region may contain important information about the reaction mechanism. However, the study of this region is outside the scope of our paper. From now on, we shall discuss only the p-shell removal. As expected from the free isospin ratios, the cross sections for the (π^-,π^-p) reaction are typically 1 order of magnitude smaller than for the (π^+,π^+p) reaction. In this respect, the theory is qualitatively consistent with the experiment. However, quantitatively the discrepancies between theory and experiment are much more severe than in the (π^+,π^+p) case. The cross section around the peak is overestimated for pion angles of 35° and 60°, but underestimated for 130°. Thus

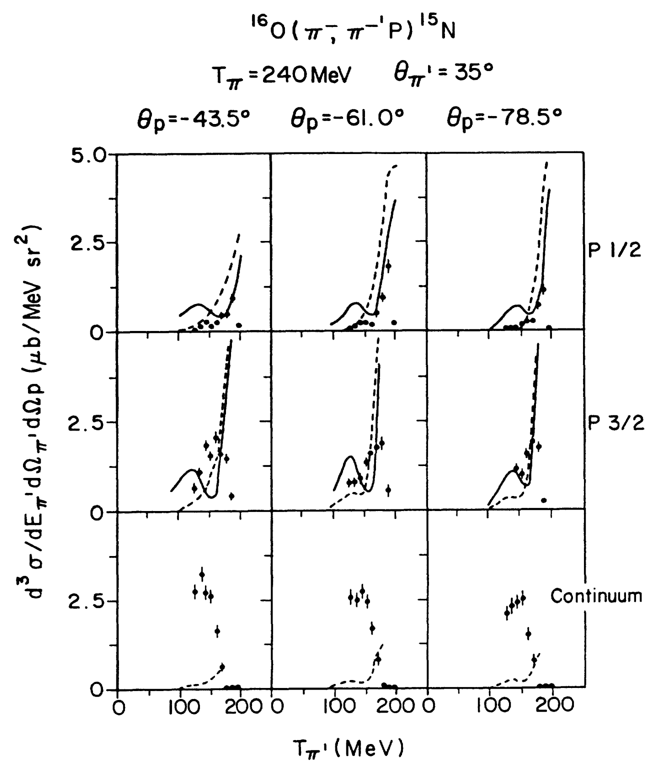

FIG. 12. Differential cross sections for ^{16}O$(\pi^-,\pi^-p)^{15}$N at 35° pion scattering angle for each of the proton telescopes, and regions of ^{15}N excitation energy corresponding to $p_{1/2}$ shell removal, $p_{3/2}$ shell removal, and continuum final state. Solid curves: full calculation with Δ-N interaction (C_{21} = 6.00-i7.09 fm^2); dashed curves: modified DWIA calculation (without Δ-N interaction). Data, from Refs. 21 and 25.

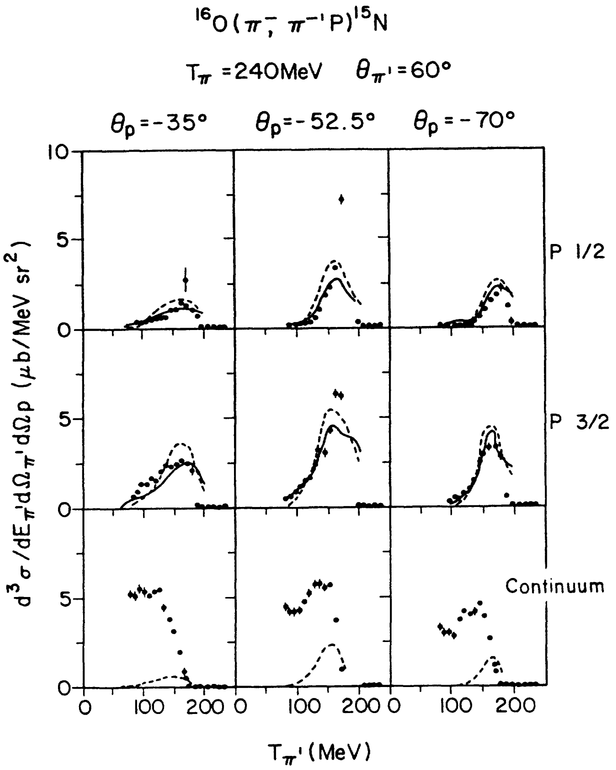

FIG. 13. Same as Fig. 12, but 60° pion scattering angle and solid curves: full calculation with Δ-N interaction ($C_{21}=2.17-i5.31$ fm^2).

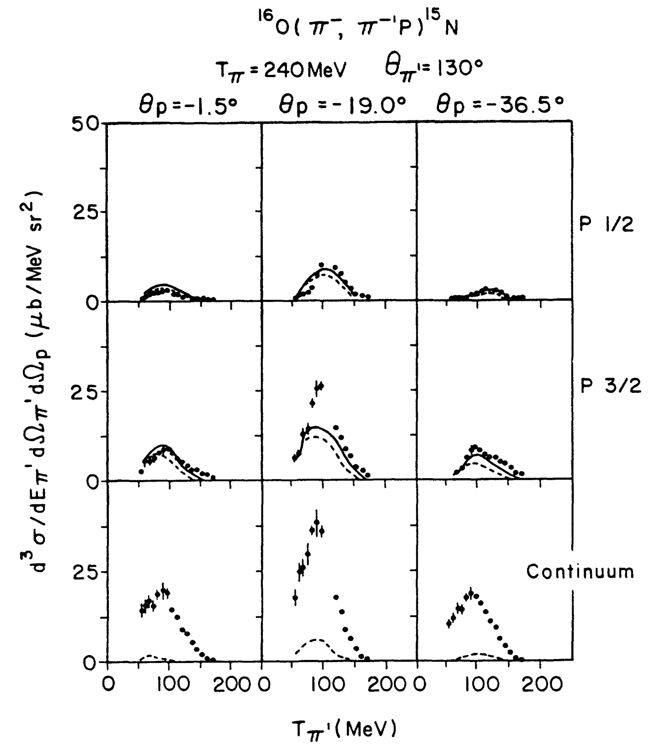

FIG. 14. Same as Fig. 12, but 130° pion scattering angle and solid curves: full calculation with Δ-N interaction ($C_{21}=-2.00-i5.31$ fm^2).

the experimental $(\pi^+,\pi^+p)/(\pi^-,\pi^-p)$ ratios differ from the free ones and cannot be understood within the DWIA framework. We shall come back to these ratios below (see Fig. 16).

We finally present the results of the (π^+,π^0p) reaction at five angles in Fig. 15. Calculations in the modified DWIA (dashed line) are compared to the LAMPF data.[22] The solid line in the figure will be discussed in the next section. The proton was detected at the quasifree angle. Here, the characteristic p-shell behavior in the energy spectra of the π^0's can be seen more clearly in the calculation than in the experiment. This discrepancy may be due to the insufficient energy resolution of the π^0. A certain contamination of s-shell removal and multiparticle emission into the energy spectra is unavoidable under the experimental conditions. A more serious problem is that the theory systematically underestimates the data at all pion angles. Consequently, the experimental $(\pi^+,\pi^+p)/(\pi^+,\pi^0p)$ ratios cannot be reproduced within the DWIA approach. Like in the case of the (π^-,π^-p) reaction, the data seem to indicate the presence of a new mechanism beyond the DWIA.

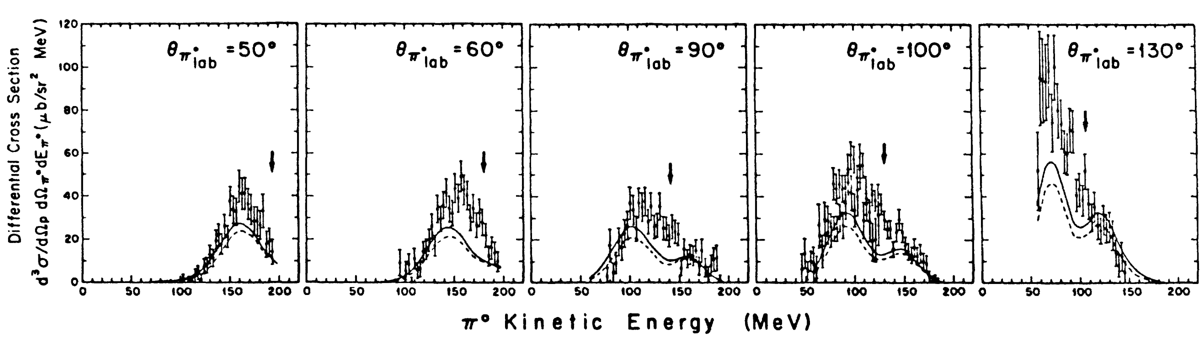

FIG. 15. Differential cross sections for $^{16}O(\pi^+,\pi^0p)$ at five pion scattering angles. Solid curves: full calculation with Δ-N interaction ($C_{21}=6.00-i7.09$ fm^2); dashed curves: modified DWIA calculation (without Δ-N interaction). Data from Ref. 22.

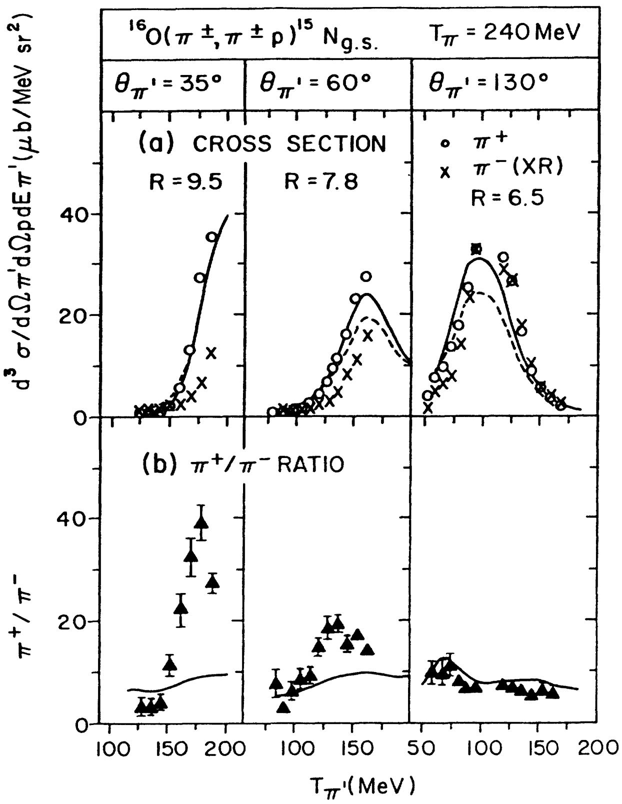

FIG. 16. (a) Differential cross sections for $^{16}O(\pi^{\pm},\pi^{\pm}p)^{15}N_{g.s.}$, averaged over three proton angles in Figs. 9–14, for each pion angle and as a function of pion energy. The π^- data are multiplied by R (the free π^+p/π^-p cross-section ratio), which is taken from Ref. 20. Solid curves: modified DWIA calculation for π^+; dashed curves: the same calculation for π^-. (b) Ratios of π^+ to π^- induced cross section for the conditions in (a). Solid curves: modified DWIA calculation. Data from Ref. 20.

The conclusions about the absolute cross sections drawn above depend critically on the use of spectroscopic factors derived from the naive shell model ($N_\alpha = 1.0$ for a closed shell). It is generally believed that the most reliable way of determinating spectroscopic factors is by means of the $(e,e'p)$ reaction. For ^{16}O, such data are available and yield $N_\alpha=0.59$ (0.57) for the $p_{1/2}$ ($p_{3/2}$) shell,[26] i.e., less than 60% of the shell model value. If we interpret these numbers as a genuine nuclear structure effect, we have to renormalize all calculated cross sections by N_α, and we underestimate the strong (π^+,π^+p) cross section by almost a factor of 2. On the other hand, it is worth recalling that as a rule, hadronic probes tend to yield spectroscopic factors closer to the shell-model expectation. For the pick-up reaction $^{16}O(d,{}^3He)$, for instance, the numbers are 1.17 (0.92) for $p_{1/2}$ ($p_{3/2}$), respectively.[27] It is natural to blame the less well-understood reaction mechanism and distortion effects in the hadronic case for this discrepancy. For pions in the resonance region, however, the theory is rather well founded, and, more importantly, very tightly constrained by other data, in particular elastic and inclusive inelastic scattering. It is not easy to think of a correction which would enhance the exclusive (π^+,π^+p) cross section by about a factor 2, without completely destroying the quantitative agreement with elastic and quasielastic scattering data. The most likely candidate, namely proton final-state interactions, can practically be ruled out as a source of drastic errors, since the same type of optical potentials has been employed in the analyses of $(e,e'p)$ and $(\pi,\pi'p)$.

It should be noted that the situation is less clear for the $p_{3/2}$ shell removal than for the $p_{1/2}$ shell. In particular, the comparison between the spectroscopic factors for the $p_{3/2}$ shell removal obtained from $(e,e'p)$ and $(\pi,\pi'p)$ is obscured by the different cuts used (3 MeV $< E_X <$ 8 MeV for electron and 4 MeV $< E_X <$ 15 MeV for pion). It is also instructive to observe the following facts: First, the (π^+,π^+p) cross section below the excitation energy of 15 MeV (i.e., approximately the threshold of the two-nucleon emission), which corresponds to the summed cross section of the $p_{1/2}$ and $p_{3/2}$ shell removal, can be explained within our simple picture of one-nucleon knockout from the p-shell states (i.e., $N_\alpha=1.0$). In this respect, the gross features of our results for the (π^+,π^+p) seem consistent with the calculation[10] of the inclusive (π,π') reaction. Secondly, such a situation can also be seen in a recent experimental study of ^{12}C $(e,e'p)$,[28] where the spectroscopic factor for the p-shell removal has been found to be nearly 1.0. In this case, many excited states in addition to the ^{11}B ground state are included in the region of the p-shell removal corresponding to the excitation energy below the threshold of the two-nucleon emission. It is important to keep in mind that the spectroscopic factor is sensitive to the cuts and the energy resolution.

However, once we confine ourselves to the ^{15}N ground state (i.e., $p_{1/2}$ shell removal), we are not able to reconcile the pion and electron scattering data for the moment. Notice that in the isospin ratios, the spectroscopic factors drop out, so that is is possible to decouple the problem of the Δ-N interaction effects from that of the nuclear structure, at least to some extent. The effects of the Δ-N interaction will be the subject of the next section.

IV. Δ-N INTERACTION EFFECTS

In the previous section we have tried to delimit the range of applicability of the modified DWIA to the $(\pi,\pi'N)$ reaction. The empirical fact that the isospin ratios strongly deviate from the free ones is a clear signal for the presence of other excitation mechanisms. In this section we consider the possibility that the nucleon is knocked out in a direct ΔN collision, and add the corresponding amplitude coherently to the modified DWIA. It should be clear that this part of our work is of more exploratory nature than the preceding one. First, in order to calculate such a process, one needs a much more detailed understanding of the Δ-nucleus interaction than in the DWIA approach, where the phenomenological spreading potential is sufficient. Secondly, the technical complications of the calculation are such that we are presently restricted to a very simple form of the Δ-N interaction. Besides, it is not practical to make extensive parameter searches for various ΔN partial waves, of the type done by Takaki[13] for discrete excitations. Our main aim is to investigate to which extent the isospin ratios is knockout reactions are modified by the Δ-N interaction governing the standard absorption process, i.e., $\Delta N \rightarrow NN$ in the ${}^5S_2(T_{\Delta N}=1)$ channel. Before turning to our numerical results calculated by using the zero-range Δ-N interaction of Eq. (22), it is instructive to exhibit the qualitative features of the Δ-N interaction effects, as first studied in Ref. 12. Following Hirata *et al.*,[12] we assume the static and closure approximation for the $\pi\Delta N$ vertex function and Δ propagator, respectively, and neglect the distortion of pion and proton. Then we obtain the following analytical form for the transition amplitudes (19) and (21) up to a common factor:

$$f_{\mathrm{I+II}}(\pi^+ \rightarrow \pi^+ pp^{-1}) \sim R_{l_h}(\mathbf{Q}) + \beta(\tfrac{1}{9}C_{21})\tilde{R}_{l_h}(\mathbf{Q}) ,$$

$$f_{\mathrm{I+II}}(\pi^- \rightarrow \pi^- pp^{-1}) \sim \tfrac{1}{3}\{R_{l_h}(\mathbf{Q}) + \beta C_{21}\tilde{R}_{l_h}(\mathbf{Q})\} , \quad (24)$$

$$f_{\mathrm{I+II}}(\pi^+ \rightarrow \pi^0 pn^{-1}) \sim -\frac{\sqrt{2}}{3}\{R_{l_h}(\mathbf{Q}) + \beta(-\tfrac{1}{3}C_{21})\tilde{R}_{l_h}(\mathbf{Q})\} ,$$

with

$$\beta = \frac{15}{16} \cdot \frac{\tilde{D}_0(E)}{\tilde{D}_1(E)\tilde{D}_2(E)} . \quad (25)$$

Here the momentum transfer **Q** is defined as

$$\mathbf{Q} = \mathbf{k} - \mathbf{k}' - \mathbf{p} . \quad (26)$$

The form factors appearing in (24) are

$$R_{l_h}(\mathbf{Q}) = \int_0^\infty r^2 dr j_{l_h}(Qr) R_{l_h}(r) ,$$
$$\tilde{R}_{l_h}(\mathbf{Q}) = \int_0^\infty r^2 dr j_{l_h}(Qr) \rho(r) R_{l_h}(r) , \quad (27)$$

where $\rho(r)$ is the nuclear density. The $\tilde{D}_i^{-1}$'s are the distorted Δ propagators which are defined in Ref. 12. $\tilde{D}_0^{-1}$ denotes the Δ propagator of the first-order term and $\tilde{D}_1^{-1}(\tilde{D}_2^{-1})$ denoted the initial (final) Δ propagator of the second-order term. From Eq. (25) one can immediately see the following qualitative feature of the Δ-N interaction effects in the ${}^5S_2(T_{\Delta N}=1)$ channel: the second-order effect is the most important for the $(\pi^-,\pi^- p)$ reaction, where it is nine times larger than for the $(\pi^+,\pi^+ p)$ reaction, relative to the first-order process. For the charge-exchange reaction $(\pi^+,\pi^0 p)$, the effective strength is $\frac{1}{3}$ of the $(\pi^-,\pi^- p)$ case but it has opposite sign. From the numerical estimate based on the analytical form (24), Hirata *et al.* concluded that the $(\pi^+,\pi^+ p)$ reaction is basically quasifree, while in the $(\pi^-,\pi^- p)$ reaction, first- and second-order terms are of equal size and interfere destructively, suppressing the magnitude of the cross section at resonance. Conversely, the $(\pi^+,\pi^0 p)$ reaction is expected to be enhanced by the effect of Δ-N interaction. Judging from the DWIA results in the previous section, both of these effects seem to go into the right direction.

Now we examine the effect of the Δ-N interaction more quantitatively. Let us first recall our modified DWIA calculations with the data of ${}^{16}\mathrm{O}(\pi^\pm,\pi^\pm p){}^{15}\mathrm{N}_{\mathrm{g.s.}}$, which corresponds to the $p_{1/2}$ shell proton removal. Figure 16 shows both the differential cross sections and the $(\pi^+,\pi^+ p)/(\pi^-,\pi^- p)$ ratios averaged over three-proton angles in Figs. 9–14, for three pion angles and as a function of pion energy. The calculation within the DWIA approach agrees well with the experiment for $(\pi^+,\pi^+ p)$, but overestimates the cross section for $(\pi^-,\pi^- p)$ at small pion angles (35° and 60°) and underestimates it at the largest angle (130°). Trivially, this calculation cannot reproduce isospin ratios which deviate strongly and in an energy dependent way from the quasifree values. We note that the weak energy dependence of the calculated ratios is due to the inclusion of the nonresonant πN interaction in the transition operator and Coulomb effects.

In Fig. 17 we present the calculations with the second-order process by the Δ-N interaction, Eq. (20), together with the same data as in Fig. 16. The strength parameter C_{21} used here is 2.17-*i*5.31 fm^2 as given in (23) and consistent with the spreading potential. In all our calculations, the transition via $\Delta N \rightarrow \Delta N$ is evaluated from the zero-range Δ-N interaction of (22), but the Δ propagators are evaluated by using the spreading potential. In principle, because of consistency, the Δ propagators have to be calculated from the Δ-N interaction instead of the spreading potential as discussed in Sec. II. However, it is known that for a zero-range Δ-N interaction like (22), the results of these two calculations would be almost indistinguishable.[13]

Confirming the simple estimates, we find that the

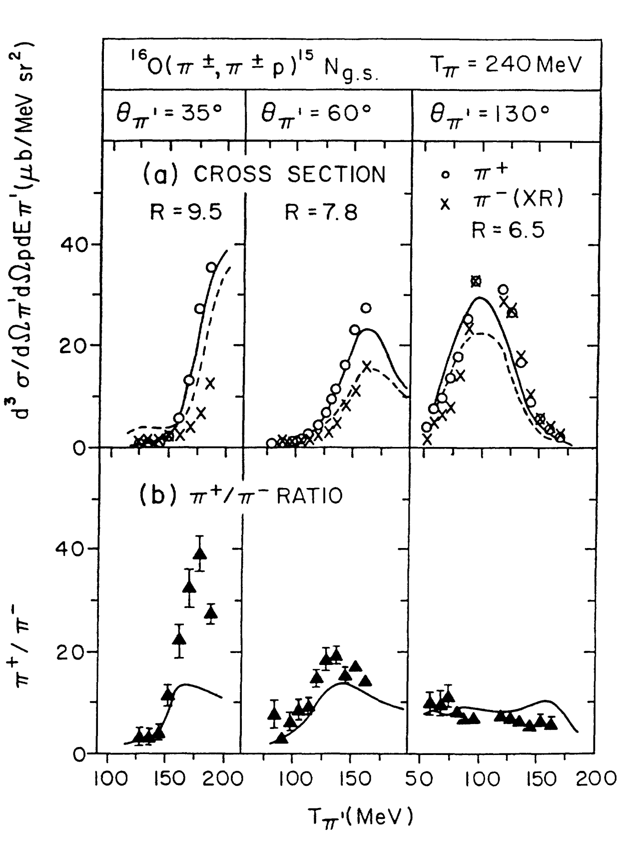

FIG. 17. Same as Fig. 16, but full calculation with Δ-N interaction ($C_{21}=2.17-i5.31$ fm^2).

second-order effect in the present calculation decreases the (π^-,π^-p) cross section, but leaves the (π^+,π^+p) cross section almost unchanged. Consequently the calculated $(\pi^+,\pi^+p)/(\pi^-,\pi^-p)$ ratio deviates from the quasifree value. Actually at the pion angle of 60°, the theory with the Δ-N interaction is in quite good agreement with the data. Particularly, the energy dependence of the ratio is described satisfactorily. The effect of the second-order process is governed by the strength parameter C_{21}, the energy-dependent factor β, and the ratio of form factors [cf. Eq. (24)]. Roughly speaking, at resonance, β tends to be negative and purely imaginary, and therefore the second-order term interfers destructively with the first-order term. This effect, which follows directly from our assumption that the Δ-N interaction arises from the strong coupling to the absorption channel, seems to be supported by the data. It should also be noted that since the quantity β depends on the final pion energy, the effect of the Δ-N interaction at the peak will be different for different pion angles. In the tail region of the energy spectra where the first-order process is suppressed, the second-order process becomes dominant, owing to the momentum sharing of the two nucleons involved in the reaction.

At the pion angle of 35° where the peak of the energy spectra is located above resonance, although the Δ-N interaction effect favorably reduces the (π^-,π^-p) cross section, the strength of the interaction is not large enough to reproduce the data. At 130°, the cross section of (π^-,π^-p) at the peak is little affected by the second-order process. From these results, we conclude that the qualitative behavior of the (π^-,π^-p) data can be understood within our model, but the theory with a single strength parameter C_{21} is unable to reproduce the data at all pion angles consistently and quantitatively. We emphasize that there are no free parameters so far in our calculations: By assuming that the Δ-N interaction is zero range and acts only in the $T_{\Delta N}=1$, $S_{\Delta N}=2$ channel, we can simply infer its strength from the spreading potential determined to fit the elastic π-nucleus data.

To improve the theoretical description of the (π^-,π^-p) reaction, it is necessary to reconsider the assumptions entering in our calculation, some of which are presumably too restrictive. At the present stage we assume that the Δ propagator is appropriately described by the spreading potential, because of the successful description of the (π^+,π^+p) reaction. However, one might question the identification between the Δ-N interaction used in the transition operator and the spreading potential. If one assumes that the spreading potential contains other effects than just $\Delta N \rightarrow NN$ absorption in one particular channel, there is no reason to strictly identify C_{21} with the value given in Eq. (23). Furthermore, we note that depending on the final pion angle, the kinematics of the ΔN collision may change appreciably. For instance, if the relative momentum $\mathbf{p}_{\rm rel}$ of the Δ-N system is assumed to be

$$\mathbf{p}_{\rm rel}=\frac{M\mathbf{p}_\Delta - M_\Delta \mathbf{p}}{M_\Delta + M}, \tag{28}$$

where $\mathbf{p}(\mathbf{p}_\Delta)$ are momenta of nucleon (Δ resonance) and M (=940 MeV) is nucleon mass and M_Δ (=1233 MeV) is Δ mass, then $|\mathbf{p}_{\rm rel}|$ becomes ~200 MeV/c at the pion angle of 35° and ~360 MeV/c at 130°. These values were calculated at the quasifree peak, and the Δ momentum was identified with the final pion momentum.

In order to study the sensitivity of our results to the precise value of C_{21}, we now allow this parameter to vary, although its imaginary part is still kept large and negative because it should represent strong absorption. Calculations with two different values of C_{21}, which were found by varying C_{21} to fit the (π^-,π^-p) data as well as the $(\pi^+,\pi^+p)/(\pi^-,\pi^-p)$ ratio, are shown in Figs. 18 and 19. Strong repulsion is required to fit the ratio with the large deviation from the quasifree value at forward pion angles. This strong repulsive interaction ($C_{21}=6.00-i7.09$ fm^2), however, produces an unrealistic bump in the tail region at forward scattering of (π^-,π^-p) and decreases the cross section of the peak at 130°, thereby worsening the agreement between theory and experiment. In order to reproduce the (π^-,π^-p) data at 130°, an attractive Δ-N interaction is needed rather than a repulsive one. However, this attractive interaction ($C_{21}=-2.00-i5.31$ fm^2) destroys the agreement between theory and experiment at forward angles. Thus the values of C_{21} obtained from fitting the data have a strong dependence on the pion angle, or equivalently, on the final relative momentum of the ΔN system.

It is interesting to see whether the calculations with the adjusted values of C_{21} reproduce the data measured in

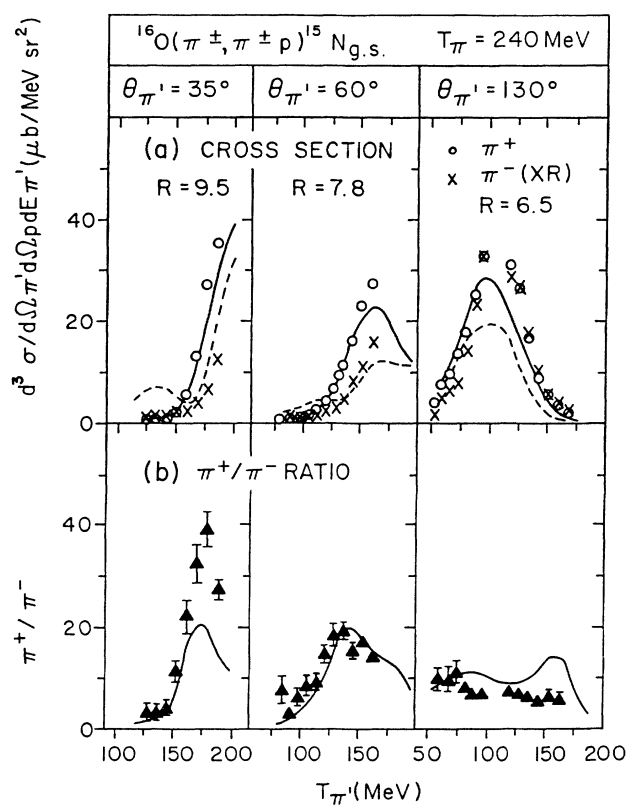

FIG. 18. Same as Fig. 16, but full calculation with Δ-N interaction ($C_{21}=6.00-i7.09$ fm^2).

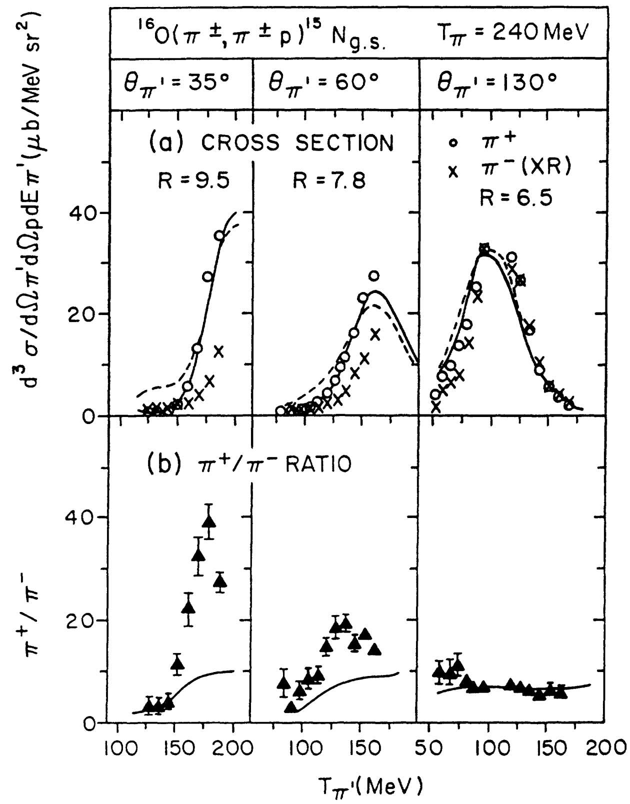

FIG. 19. Same as Fig. 16, but full calculation with Δ-N interaction ($C_{21}=-2.00-i5.31$ fm^2).

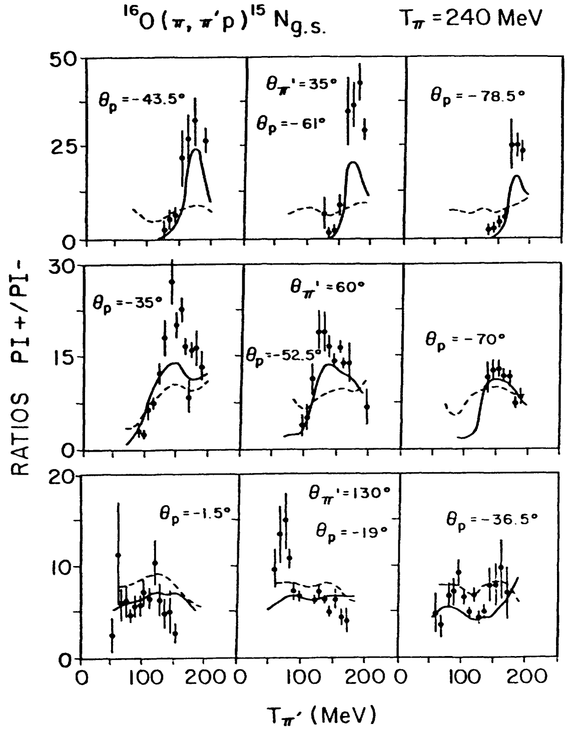

FIG. 20. Ratios of differential cross sections for $(\pi^+,\pi^+p)/(\pi^-,\pi^-p)$ for $p_{1/2}$ shell removal. Pion scattering angles are 35°, 60°, and 130°; the corresponding proton angles are the same as those in Figs. 9–14. Dashed curves: modified DWIA calculation; solid curves: full calculation with Δ-N interaction. The strength parameter C_{21} used is $6.00-i7.09$ fm^2. $2.17-i5.21$ fm^2 $-2.00-i5.31$ fm^2 for 35°, 60°, and 130°, respectively. Data from Ref. 21.

each of the three proton detectors, i.e., the data which have already been presented in Figs. 9–14. The calculated cross sections of the (π^-,π^-p) reaction are shown in Figs. 12, 13, and 14 (solid lines). The calculated $(\pi^+,\pi^+p)/(\pi^-,\pi^-p)$ ratio for the $p_{1/2}$ shell removal is also shown in Fig. 20, together with the data. For the (π^+,π^+p) reaction, the effect of the second-order process is at most 10%, so that the full calculation is almost the same as the DWIA in Figs. 9–11. It is seen that the theory agrees reasonably with the data at all three angles except in the region of low-energy pion emission at 35°, once we allow angular dependence of C_{21}. This angular dependence could indicate that either the zero-range approximation, or the restriction to ΔN s waves is too crude. As pointed out before, there is a large difference in the relative momentum of the ΔN system between the pion angles of 35° and 130°.

Finally we discuss the effect of the Δ-N interaction on the charge exchange reaction (π^+,π^0p). We recall that the second-order process modifies the (π^+,π^0p) cross section in the opposite direction to the (π^-,π^-p) cross section, but its effective strength is a factor of 3 smaller. From the above qualitative considerations and the (π^-,π^-p) results, we expect that a strong repulsive Δ-N interaction is needed at all angles in order to improve the agreement with the (π^+,π^0p) data, since the calculations without the second-order process underestimate the data significantly. The calculation with $C_{21}=6.00-i7.09$ fm^2, i.e., the value found by fitting the (π^-,π^-p) data at the pion angle of 35°, are shown in Fig. 15. We find that the second-order process enhances the cross sections by $\sim$20% and improves the agreement with the data, but a systematic overall disagreement remains. This is reminiscent of a similar shortcoming of the Δ-hole model in the description of single charge exchange in ^{13}C to the isobaric analogue state (cf. Ref. 29). Hence it does not seem possible to consistently explain all three isospin channels in the $(\pi,\pi'N)$ reaction within our model, including only the $T_{\Delta N}=1$, $S_{\Delta N}=2$ s-wave Δ-N interaction.

V. SUMMARY AND CONCLUSIONS

In this work we have presented first systematic study of the $(\pi,\pi'N)$ knockout reaction in the framework of the Δ-hole model. Although the advantages of this reaction were well known long ago, coincidence data with good energy resolution and sufficient statistics have only become available during a last few years. The fact that all three isospin channels have been measured for ^{16}O near 240 MeV makes a combined, quantitative analysis desirable and timely. However, it is also clear from the anomalous isospin ratios that one has to go beyond the DWIA in order to have any chance to explain the data. This poses great technical problems on top of the usual difficulties inherent in Δ-hole calculations, while increasing the uncertainties in the reaction mechanism and in the dynamical input.

In Sec. II we have reviewed the Δ-hole formalism and pointed out the role of the Δ-N interaction. The most important new feature as compared to elastic scattering is the fact that one has to go to a more microscopic level than that of the spreading potential, as pointed out by Hirata *et al.*[12] An attractive possibility is to parametrize the effective ΔN t matrix, rather than the Δ-nucleus potential. For elastic scattering, this yields nothing new, but for inelastic reactions the implications are quite striking and show the potential interest of these studies, even if the present state of the art is still too crude.

In Sec. III we have presented results of a calculation within the modified DWIA. By this we mean that the transition operator is not the free πN t matrix, but contains the same medium corrections as in elastic scattering. This approach has been successfully used for the inclusive (π,π') reaction before, and has the advantage that it is free of parameters, once the spreading potential has been extracted from elastic scattering. Not surprisingly, such a calculation yields isospin ratios which are close to the free ones, and therefore at variance with the experimental data. Nevertheless, it already provides some important information about the absolute magnitude of the predicted cross sections. When using spectroscopic factors given by the shell model, we found that the data for the strong (π^+,π^+p) channel are generally well reproduced, whereas severe discrepancies show up in the weaker (π^-,π^-p) and (π^+,π^0p) channels. This looks perfectly consistent with the results of the inclusive (π,π') reaction, which is dominated by (π^+,π^+p) and differs from the exclusive reaction mainly through (moderate) proton final-state interaction effects. However, we have to face the disturbing features that $(e,e'p)$ measurements seem to indicate much smaller spectroscopic factors (by 40% in our case), and we have commented on the difficulty to reconcile all the known facts, if the final state is restricted to the ^{15}N ground state or the $p_{1/2}$-hole state. If it is true that NN correlations in the ground state are responsible for the small spectroscopic factors seen with electrons, one cannot rule out that these correlations would play a different role in hadronic reactions, where multiple scattering is important. Such correlation effects have never been considered within the Δ-hole model, so that we do not know to which extent they would upset the present phenomenology. Our conclusion is that by using the same simple nuclear structure input in the analyses of elastic π-nucleus scattering and the one-nucleon knockout reaction, we seem to obtain a consistent picture for the dominant (π^+,π^+p) process.

In Sec. IV we have focused our attention on the *active* role of the Δ-N interaction, that is the proton knockout induced by ΔN collisions. Here, we have restricted ourselves to very simple type of interaction, primarily in order to avoid the introduction of too many unconstrained parameters. Our basic working hypothesis was that the Δ-N interaction is dominated by the $^5S_2(T_{\Delta N}=1)$ channel. When fixing the strength parameter via the empirical spreading potential, we found that the (π^+,π^+p) reaction was only little affected, whereas the (π^-,π^-p) cross section was decreased and the single charge exchange somewhat increased. This confirms nicely earlier estimates, although we have not come much closer to a quantitative understanding of the reaction. Since the assumption of $^5S_2$ dominance is safer for the imaginary part than for the real part of the effective Δ-N interaction, we have allowed the real part of C_{21} to vary freely. Even so, it was found necessary to allow for a strong dependence of the strength parameter on the pion scattering angle in order to reproduce the (π^-,π^-p) data, a possible hint that our Δ-N interaction is too simple. For the charge exchange, we found that the Δ-N interaction cannot provide enough enhancement. The residual discrepancy between theory and experiment is of the same order as the one found in the $^{13}\mathrm{C}(\pi^+,\pi^0)^{13}N_{\mathrm{g.s.}}$ reaction.[29] We can practically rule out that this defect may be cured if we restrict ourselves to a pure $^5\mathrm{S}_2(T_{\Delta N}=1)$ Δ-N interaction.

To arrive at the above conclusions, we have employed several assumptions in this work: (a) the reaction takes place dominantly through the Δ resonance, (b) the spreading potential is constructed from an effective two-body Δ-N interaction, (c) this effective Δ-N interaction is generated by the coupling to the pion absorption channel, (d) the Δ-N interaction is represented by a zero-range interaction in the $T_{\Delta N}=1$, $S_{\Delta N}=2$ channel. We have already questioned the assumption (d). The next step would be to take into account finite-range effects and higher partial waves (especially p wave). If Δ-N scattering is also quasifree at the quasifree peak of the $(\pi,\pi'p)$ reaction, the Δ would scatter forward (backward) corresponding to forward (backward) pion scattering. If this process is described by the following s wave Δ-N interaction,

$$\tilde{t}_{\Delta N}=v(p'_{\mathrm{rel}})\lambda(E)v(p_{\mathrm{rel}}) , \tag{29}$$

with

$$v(p_{\mathrm{rel}})=\frac{1}{p_{\mathrm{rel}}^2+m_\pi^2} , \tag{30}$$

where the relative momentum of ΔN, $\mathbf{p}_{\mathrm{rel}}$, is defined in Eq. (28) and $\lambda(E)$ is an energy-dependent strength parameter of the interaction, we would expect that the ratio of the effective strength of the Δ-N interaction between pion angles 35° and 130° is roughly 2.6. Although it is not clear whether the form (29) is adequate, this crude estimate suggests the necessity of a study using a refined model for the Δ-N interaction.

The finite-range effect as well as the p-wave interaction effect might give rise to the angular dependence of the Δ-N interaction which seems to be required by the (π^-,π^-p) data. However, a more serious problem is that the theory underestimates the (π^+,π^0p) reaction significantly. The above effects do not help to solve this problem.

Under the assumptions of (a), (b), and (c), one is inevitably led to pay attention to the $T_{\Delta N}=2$ Δ-N interaction. Although a pure $T_{\Delta N}=2$ interaction cannot change the isospin ratios, it will have an effect in the presence of a $T_{\Delta N}=1$ component. The importance of the $T_{\Delta N}=2$ interaction is due to the fact that it might be related to the multinucleon absorption mechanisms. In fact, recent studies of pion absorption[30] have shown that the two-nucleon absorption mechanism, which generates the $T_{\Delta N}=1$ Δ-N interaction, cannot explain the total absorption cross section. In view of this situation, it seems to use that further progress towards a quantitative understanding of the knockout reactions has to await a better understanding of pion absorption.

ACKNOWLEDGMENTS

We thank C. H. Q. Ingram and G. S. Kyle for providing us with Swiss Institute for Nuclear Research (SIN) data and for useful discussions. We are grateful to F. Lenz for valuable discussions and his encouragement during this work. We also thank E. J. Moniz for a critical reading of the manuscript. One of us (T.T.) acknowledges the hospitality at SIN, where the numerical calculations were mostly done. This work is supported in part by SIN, by funds provided by the U.S. Department of Energy (DOE) under Contract No. DE-AC02-76ER03069, and by the Foundation for Fundamental Research on Matter (FOM) and the Netherlands Organization for the Advancement of Pure Research (ZWO).

APPENDIX

We would like to discuss briefly the rearrangement process shown in the diagram (part 4) in Fig. 7. This diagram is obtained by the exchange of a hole line in the diagram (part 2), which corresponds to the lowest order process for final pion distortion. As this rearrangement process arises from one pion exchange W_π, the contribution of the $T_{\Delta N}=2$ component in the interaction is three times larger than the $T_{\Delta N}=1$ component. The amplitudes for the first- and second-order processes are written in terms of the (π^+,π^+p) amplitude, if the nonresonant transition and the Coulomb force is neglected, as

$$\begin{aligned} f_{\mathrm{I+II}}(\pi^+\to\pi^+pp^{-1})&=f_{R_0}+f_{\mathrm{Re}}\ ,\\ f_{\mathrm{I+II}}(\pi^-\to\pi^-pp^{-1})&=\frac{1}{3}(f_{R_0}+\tfrac{3}{11}f_{\mathrm{Re}})\ ,\\ f_{\mathrm{I+II}}(\pi^+\to\pi^0pn^{-1})&=-\frac{\sqrt{2}}{3}(f_{R_0}+\tfrac{15}{11}f_{\mathrm{Re}})\ . \end{aligned} \tag{A1}$$

Here $f_{\mathrm{Re}}(\pi^+,\pi^+p)$ denotes the amplitude of the diagram (Fig. 7, part 4). From this expression, one can see that this second-order process is most important for the (π^+,π^0p) reaction, whereas its effect on the (π^-,π^-p) reaction is a factor of 5 smaller. A crude estimate indicates that the rearrangement process modifies the first-order process by only 10–20 % for the $^{16}O(\pi^+,\pi^0p)$ reaction at the quasifree peak. We should note that for ^4He, the amplitude of this second-order process is just half of that of the diagram for the distortion (Fig. 7, part 2) and so this process is expected to affect the cross section significantly. However, the magnitude of the correction on ^{16}O is still not enough to reproduce the charge exchange data. When further detailed study on the charge exchange reaction is pursued, a reliable calculation of this rearrangement process is needed in spite of the computational difficulties.

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